

Debunking the Economic Order Quantity Concept

The concept of Economic Order Quantity (EOQ) describes the amount a supply house or distribution firm should next procure from a manufacturer or vendor so that profit will be maximized. The relationships and mathematics for describing this concept seem to be straightforward, yet there are several wrinkles not considered by the formulas given. Additionally, those in the industry are quick to provide their equations without derivation or concern for these wrinkles. Finally, although possibly workable for a firm having only one product, the demands of having a finite warehouse space shared by hundreds or thousands of products make the use of the concept of EOQ unwise there, especially when other methods of estimating purchases are available.

The relationships for single and multiple products are laid out in our textbook, *Managerial Economics*, Keat & Young, in Chapter 7. The optimal order quantity would be found when Marginal Cost equals Marginal Revenue equals zero. Marginal Revenue is the derivative of the Total Revenue equation. Using this math, I arrive at the following computations for supply/distribution house inventory.

$$TR = P * Q$$

$$pi = TR - TC$$

where pi=total profit

TC=carrying cost + purchasing cost + fixed costs

So $pi = PQ - (\text{carrying cost} + \text{purchasing cost} + \text{fixed costs})$

where carrying cost and purchasing cost are functions of Q

I estimate PC (purchasing cost) to be a linear function,

$$PC = (\text{unit cost of item} + \text{procurement cost})Q$$

(beware of quantity breaks, so unit cost of item is a $f(Q)$)

CC (carrying cost) would have the following factors,

$$CC = lL + rK$$

Where l =wage rate of labor, L =amount of Labor (an $f(Q)$)

Where r =rental cost of using capital, K =Capital (an $f(Q)$)

(Here I differ from the text because I disagree that rK is a continuous function. Calculus may only be applied in the short run, the long term jumps when new warehouse space is acquired.)

$$rK = uW + \text{overhead}$$

(Note that u =cost of warehouse space, W =amount of space (an $f(Q)$). Note also that this means the cost is a function of the warehouse space used, important later.)

$$\frac{\partial pi}{dq} = \frac{P \partial pi}{\partial q} + \frac{CC \partial pi}{\partial q} + \frac{PC \partial pi}{\partial q}$$

This is where the whole thing breaks down. Since warehouse space is finite in the short term and multiple products vie for that warehouse space, the ones evaluated first would tend to take up all of the warehouse space. There are many combinations of products and order quantities that could produce an optimal profit in the short run with limited warehouse space. A prioritization system would be necessary for the proper functioning of such a set of calculations, where one would decide from the outset which products one favors, i.e. which define the business most closely, but this partially defeats the purpose of having a maximization system in place for multiple products.

One other place it breaks down is in the concept of Loss Leaders. These are products whose cost of carrying is high and margins low or actually negative that are

carried and actively sold because they draw people into the store. These are often necessities and sometimes specials. They attract business and bring profit because the complements of the goods are sold and are profitable and sometimes they generate business for other products altogether. Few or none of these Loss Leaders would be ordered by such a calculation.

Some attempts have been made to actually fully define the equations that go into the Total Cost formula. One such equation has been put forth by a self-proclaimed inventory expert and author of a couple of books on inventory in distribution. His name is Gordon Graham and is the author of “Distribution Inventory Management for the 1990s” and “Distributor Survival in the 21st Century”. In the first of these, he graphically presents the cost and quantity curves and the EOQ point, but then goes on to proclaim the following formula, without derivation, for the whole industry to use.

$$EOQ = \sqrt{\frac{(24 \times PC \times \text{demand})}{(CC \times \text{unit cost})}}$$

Unfortunately for those using Gordon’s equations, the CC function is in the denominator, which, from our discussion above would seem to be incorrect. Indeed, in previous attempts to derive the functions for this calculation, I have found that the powers of Q for CC are always of the same order as the powers of Q of the PC which would indicate that they should both be in the numerator. I have made several such attempts, but always found that the equations had many factors, most of which were incalculable or at least would require a company-wide, Herculean effort to discover. Finding the purchasing and carrying cost per unit quantity(!) is very difficult when faced with an inventory comprised of many items. Also, his EOQ formula would have to be derived by calculus and he doesn’t state the need for calculating the second derivative and

determining its sign, which could leave you at a maximum cost point rather than a minimum cost point or a minimum profit point rather than a maximum profit point (he doesn't state whether it is a cost or profit equation).

Ultimately, however, the cost function is a step function and is not a continuous function. The cost function is a step function because inventory carrying costs do not rise incrementally with additional inventory but chiefly when new warehouse space is acquired, in bulk. Since continuity is necessary for calculus integration and derivation, any such calculus is invalid for the cost equation. Thus, I have discarded this equation.

This is not to say that there are no tools available to the distribution company for estimating purchases automatically based on moving averages of past demand (or even expected demand), lead times, minimum order quantities, etc. These types of things can be applied to many-item inventories by using computers. Please see the Appendix for Computer Source Code on this subject.

In order to estimate future sales, past demand figures are almost always necessary. Care must be taken to keep records at the level at which one's business functions. If one's business is slow to change and is not seasonal in nature, keeping tallies by month per product is usually sufficient. If the business is more volatile, weekly tallies are called for. Another quantity of which to be aware is how many years data to keep. If the business is seasonal, at least several years is recommended, so that demand from past years during the same period may be averaged. Also, growth factors may be worked if nonhistoric growth is expected, but, most importantly, lost sales data must be captured so that demand is tracked, not sales. Without lost sales data, items that could be sold but are

often out of stock are not replenished because they appear to be zero sales items in the figures.

Lead times are another factor in generating an appropriate order amount, since inventory will deplete during the time the shipment is on order and being delivered. Thus additional inventory must be procured to cover lead times. Likewise, allocations of stock for future delivery or pickup must be deducted from present inventory levels lest inventory be overstated. Care should be taken with this to ensure that only those allocations that are committed between the order time and, not this order's arrival, but the next order's arrival are deducted from inventory.

Minimum order quantities must be considered when placing the order for each product. When your inventory calls for 6 but the minimum order quantity by the vendor is 10, 10 must be ordered or the company must wait until the inventory depletes further. Likewise, vendor package quantities must be considered. If the vendor only ships in packages of 12, but one's inventory requires 20, one's order must be 24 (2 packages) to cover inventory requirements.

Substitutions may be considered when the vendor reports that the desired purchase item is not in stock. Alternately, backorders can be used.

With these, an order can be made. The textbook, on pgs. 245-247, expresses the moving average calculation as

$$E = \frac{(X_t + X_{t-1} + \dots + X_{t-N+1})}{N}$$

Where E=Forecast

X_t, X_{t-1} = Actual values at their respective times

N = Number of Observations included in the average

The textbook states that this is only applicable to nonseasonal goods, but I disagree. If the period is the same during each year, the result may be calculated based on an average of the previous years,

$$E = \frac{(X_{y1} + X_{y2} + \dots + X_{yn})}{n}$$

Where E=forecast

$X_{y1} \dots X_{yn}$ = elements from years y_1 through y_n in the same period in each year.

n = number of years

In fact, one can take it a step further to estimate seasonality by combining several terms from each year for several years, like

$$E = \frac{(X_{y1w1} + X_{y1w2} + X_{y1w3} + \dots + X_{y2w1} + X_{y2w2} + X_{y2w3} + \dots + X_{ynw1} + X_{ynw2} + X_{ynw3} + \dots + X_{ynwm})}{(n * m)}$$

Where E=forecast

$X_{y1w1} \dots X_{ynwm}$ = elements from m weeks in n years.

These moving average methods for forecasting demand can be applied to a database of inventory items that include both seasonal and nonseasonal goods. The growth factor would be a simple multiplicative factor that would be added to each forecast, such as

$$E_n = E_o + (E_o * \text{Growth Factor})$$

The computer source code included in the appendix is for such a database.

Lead time represents the time between ordering and receiving the product. These can be calculated from past purchase orders and averaged like demand. But, in a computer system, they are often manually entered because whole orders can be delayed due to one or two items holding them up that aren't always on the order, thus distorting the lead times.

Minimum order quantities must be applied to each product order. It is simply an override, but requires that there actually be such a quantity for a product to apply it, so that, if the minimum order quantity is zero, the original quantity is ordered.

Vendor package quantities also are only applied when such a quantity exists. If zero, then the original quantity is ordered, otherwise order a multiple of the vendor package quantity.

With substitutions, care must be taken to avoid pitfalls where the vendor package quantities are different or they ship in gallons instead of liters, for example. Substitutions should be reviewed prior to sending the order out.

In conclusion, it would seem that the simpler methods such as moving average forecasting are better when dealing with complex subjects such as many-item inventories than are Economic Order Quantity calculations. A corollary to this is that a business might be more stable and easier to forecast if it deals with one or just a few products. Being in the supply or distribution business leaves one in the unenviable position of having to stock a wide variety of goods for everyone or no one will bother to conduct business with the company. Care must be taken when applying formulas to one's business whose derivations are not shown and that one cannot verify mathematically, even if published in a book.

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